Evaluating an Interest Using the Limit

Recall that the formula for *compound interest* is:

$$A = P\left(1 + \frac{r}{k}\right)^k$$

and the anual percentage rate is:

$$\text{APR} = \left(1 + \frac{r}{k}\right)^k - 1.$$

Here P is the principal invested, r is the annual "simple" interest rate, A is the amount in the account at a given time, and k determines the frequency with which interest is added to the account.

As k approaches infinity interest is added more and more often; in the limit we say that the interest is *compounded continuously.*

1. Use the fact that $\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n = e$ to compute the APR of 5% compounded continuously.
2. Compute the APR of 10% compounded continuously.

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$$APR = \left(1+\frac{r}{k}\right)^k - 1$$

1. $$\lim_{k\to\infty}\left(1+\frac{0.05}{k}\right)^k$$

$$\frac{0.05}{k} = \frac{1}{\ell}$$

$$\ell = 20k$$

$$\Rightarrow \lim_{k\to\infty}\left(1+\frac{0.05}{k}\right)^k$$

$$= \lim_{\ell\to\infty}\left(1+\frac{1}{\ell}\right)^{\frac{\ell}{20}}$$

$$= e^{\frac{1}{20}}$$

$$\therefore APR = e^{\frac{1}{20}} - 1$$

2. $$APR = \lim_{k\to\infty}\left(1+\frac{0.10}{k}\right)^k - 1$$

$$= \lim_{\ell\to\infty}\left(1+\frac{1}{\ell}\right)^{\frac{\ell}{10}} - 1$$

$$= e^{\frac{1}{10}} - 1$$